Double, Triple and Hidden Charm Production in the Statistical Coalescence Model

A.P. Kostyuk
Institut für Theoretische Physik and Frankfurt Institute for Advanced Studies, J.W. Goethe-Universität, Frankfurt am Main, Germany
and
Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine

The production of particles with double, triple and hidden charm in heavy ion collisions is studied in the framework of the statistical coalescence model. According to the postulates of the model, the charm quark-antiquark pairs are created at the initial stage of a heavy ion reaction in hard parton collisions. The amount of charm is assumed to be unchanged at later stages. The charm (anti)quarks are distributed among different hadron species at hadronization according to the laws of statistical physics. Several approaches to the statistical treatment of charm hadronization are considered. The grand canonical approach is appropriate for systems containing large number of charm (anti)quarks. The exact charm conservation and Poissonian fluctuations of the number of charm quark-antiquark pairs should be taken into account, if the average number of these pairs is of oder of unity or smaller. The charm hadronization in a subsystem of a larger system is discussed. It is explained why the canonical approach is not appropriate for the description of charm hadronization. The obtained formulas can be used to calculate the production of charm in heavy ion collisions in a wide energy range.

PACS numbers: 25.75.-q, 14.65.Dw, 12.40.Ee, 25.75.Dw

I. INTRODUCTION

The thermal hadron gas (HG) model has demonstrated an obvious success in describing the chemical composition of light-flavored hadrons produced in heavy ion [1] and even in elementary hadron-hadron [2] as well as in electron-positron [3] collisions. The experimental data can be well fitted with only three free parameters: the temperature T, the volume V and the baryonic chemical potential μ_b of the hadron gas at the point of the chemical freeze-out. (Sometimes the fit is improved by introducing one more parameter — the strangeness suppression factor γ_s.) This success motivated attempts to extend the applicability domain of the thermal model also to heavy-flavored hadrons, for instance, to the description of J/ψ meson production [4].

A straightforward application of the equilibrium HG model to hadrons with open and hidden charm is not, however, justified. Partons with rather large momenta are needed to produce a heavy quark-antiquark pair. Consequently, the time of charm equilibration in a thermal hadronic or even quark-gluon medium is large and definitely exceeds the lifetime of the fireball. Production of charm can take place only at the initial stage of the heavy ion reaction, when hard partons are available. The charm production/annihilation rate is too low to keep the number of heavy quark-antiquark pairs at its chemical equilibrium value at later stages. Therefore, the *total amount* of heavy flavor should be out of equilibrium at the point of hadronization and chemical freeze-out. It is reasonable to expect, however, the *distribution* of heavy quarks and antiquarks among different hadrons with open and hidden charm to be thermal and controlled by the same values of thermodynamic parameters that fit the chemical composition of light-flavored hadrons. These ideas are implemented in the statistical coalescence model (SCM) [5, 6, 7].

In the present paper, I consider SCM, which is based on the following postulates:

- The charm quarks, c, and antiquarks, $\overline{c}$, are created at the initial stage of A+A reaction in hard parton collisions.
- Creation and annihilation of $c\overline{c}$ pairs at later stages can be neglected.
- The formation of observed hadrons with open and hidden charm takes place near the point of chemical freeze-out in accordance with the laws of statistical physics.

This approach appeared to be quite successful describing the J/ψ and ψ' production in (semi)central Pb+Pb collisions at Super Proton Synchrotron (SPS) [8, 9]. Although the role of the statistical coalescence at SPS energies is still under discussion [10], its dominance at the energies of the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) is more certain. The SCM-based predictions for the quarkonium production at RHIC [11, 12] are consistent with transport calculations [13].

Quarkonia are not the only type of hadrons whose production can be described by the statistical coalescence model. If two, three or more charm quark-antiquark pairs are created in a nucleus-nucleus collision, hadrons with double and triple charm can be formed. These hadronic states were long time ago predicted by the quark model (see [14] and references therein), but most of them have

not been observed experimentally yet.[1]

The hadrons with double and triple charm are of special interest from both theoretical end experimental point of view. Due to a rather large mass of charm (anti)quarks, their interactions within a hadron are close to the perturbative regime of Quantum Cromodynamics (QCD). A study of the properties of doubly and especially triply charmed baryons would allow to test QCD-based models of quark forces. An observation of more exotic hadronic states, like multi-charm tetra- and pentaquarks will open a new window into the structure of the hadronic matter. The intention to detect the double and triple charm poses a new challenge to the experimentalists and will demand a further development of the experimental technique.

In the present paper, I derive the formulas that allows to calculate the yield of double and triple charm particles. The formulas for the hidden charm are also included as their detailed derivation has not been published yet.

The article is organized as follows. In the Section II I consider the grand canonical approach. A system with exactly fixed numbers of charm quarks and antiquarks is studied in Section III. A more realistic situation, the system with Poissonian fluctuations of the number of charm quark-antiquark pairs, is considered in Section IV. Section V is devoted to the case, when only a part of the total system is available for the observation: the double, triple and hidden charm yield in a subsystem is studied. Summary is given in Section VII.

II. THE GRAND CANONICAL APPROACH TO THE STATISTICAL COALESCENCE

The grand canonical version of the statistical coalescence model was proposed in [5]. It can be applied to the systems containing large ($N_{c\bar{c}} \gg 1$) number of $c\bar{c}$ pairs.

Let us start from the grand canonical partition function for the ideal hadron gas in the Boltzmann approximation:

$$\mathcal{Z}(V,T,\{\lambda\}) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\ldots\sum_{i_L=1}^{\infty}\prod_{l=0}^{L}\frac{[\lambda_l\phi(T,m_l,g_l)V]^{i_l}}{i_l!}, \tag{1}$$

where L is the total number of hadron species (including resonances), V and T are the volume and temperature, $\phi(T;m,g)$ is the one-particle partition function

$$\begin{aligned}\phi(T;m,g) &= \frac{g}{2\pi^2}\int_0^{\infty} p^2 dp\ \exp\left(-\frac{\sqrt{p^2+m^2}}{T}\right) \\ &= g\frac{m^2T}{2\pi^2}\ K_2\left(\frac{m}{T}\right)\ .\end{aligned} \tag{2}$$

Here m is the particle mass, g is the degeneration factor (the number of spin states), K_2 is the modified Bessel function. In the nonrelativistic limit $m >> T$ the expression (2) takes the form

$$\phi(T;m,g)\ \simeq\ g\left(\frac{mT}{2\pi}\right)^{3/2}\exp(-m/T). \tag{3}$$

The fugacity λ_l is expressed via the chemical potentials (electric — μ_e, baryonic — μ_b, strangeness — μ_s and charm — μ_c) and suppression (enhancement) factors (for strangeness — γ_s, and charm — γ_c):

$$\lambda_l\ =\ \gamma_s^{|s|_l}\gamma_c^{|c|_l}\exp\left(\frac{\mu_l}{T}\right), \tag{4}$$

$$\mu_l\ =\ q_l\mu_e + b_l\mu_b + s_l\mu_s + c_l\mu_c\ , \tag{5}$$

where q_l, b_l, s_l, c_l, are, respectively, the electric charge, baryon number, strangeness and charm of the hadron species l, $|s|_l$ and $|c|_l$ are the numbers of valence strange and charmed (anti-)quarks. The chemical potentials in the right-hand-side of (5) are responsible for keeping the correct *average* values of the corresponding charges in the system. The suppression (enhancement) factors γ_s and γ_c are introduced to take into account a deviation of the total number of strangeness and charm from their equilibrium values[2].

The average number of particles is given for each species as

$$N_l = \lambda_l\frac{\partial\log\mathcal{Z}(V,T,\{\lambda\})}{\partial\lambda_l} = \lambda_l\phi(T,m_l,g_l)V. \tag{6}$$

The total number of hadrons

$$N_{tot} = \sum_l N_l,\ (l \text{ runs over all hadron species}), \tag{7}$$

can be broken up into several pieces: N_0, the number of zero charm hadrons (excluding hidden charm), N_H, the number of hidden charm mesons, N_1, $N_{\bar{1}}$, N_2, $N_{\bar{2}}$, N_3, $N_{\bar{3}}$, the numbers of hadrons with, respectively, single, double and triple charm and anticharm:

$$N_{tot} = N_0 + N_H + N_1 + N_{\bar{1}} + N_2 + N_{\bar{2}} + N_3 + N_{\bar{3}}. \tag{8}$$

Let us consider a system containing *in average* N_c charm quark and $N_{\bar{c}}$ charm antiquarks. Then, as far as charm creation and annihilation are neglected, the following equalities should be satisfied:

$$\langle N_c\rangle\ =\ N_1 + N_H + 2N_2 + 3N_3, \tag{9}$$

$$\langle N_{\bar{c}}\rangle\ =\ N_{\bar{1}} + N_H + 2N_{\bar{2}} + 3N_{\bar{3}}. \tag{10}$$

[1] The recent observation of the doubly charmed baryon Ξ_{cc}^{+} [15] is still under discussion [16].

[2] The suppression (enhancement) factor γ_s (the same for γ_c) is equivalent to an additional chemical potential $\mu_{|s|}$ which, in contrast to μ_s, has the same influence on the strangeness as on the antistrangeness: $\gamma_s = \exp\left(\frac{\mu_{|s|}}{T}\right)$

It is easy to see from (4),(5) and (6) that the above equations can be rewritten as

$$\langle N_c\rangle = \gamma_c\lambda_c\tilde{N}_1 + \gamma_c^2\tilde{N}_H + 2\gamma_c^2\lambda_c^2\tilde{N}_2 + 3\gamma_c^3\lambda_c^3\tilde{N}_3, \quad (11)$$

$$\langle N_{\bar{c}}\rangle = \gamma_c\lambda_c^{-1}\tilde{N}_{\bar{1}} + \gamma_c^2\tilde{N}_H + 2\gamma_c^2\lambda_c^{-2}\tilde{N}_{\bar{2}} + 3\gamma_c^3\lambda_c^{-3}\tilde{N}_{\bar{3}}, \quad (12)$$

where

$$\tilde{N}_k = N_k|_{\mu_c=0,\gamma_c=1}, \quad k = H, 1, \bar{1}, 2, \bar{2}, 3, \bar{3}, \quad (13)$$

and

$$\lambda_c = \exp\left(\frac{\mu_c}{T}\right). \quad (14)$$

The constituent charm quark mass is about an order of magnitude larger than the typical temperature of chemical freeze-out. From this reason,

$$\tilde{N}_1, \tilde{N}_{\bar{1}} \gg \tilde{N}_H, \tilde{N}_2, \tilde{N}_{\bar{2}} \gg \tilde{N}_3, \tilde{N}_{\bar{3}} \quad (15)$$

due to the exponential factor in (3). Therefore, if the factor γ_c is not extraordinary large, the terms in (11) and (12) corresponding to hidden, double and triple (anti)charm can be neglected in a zero approximation and the coupled equations (11) and (12) can be simplified:

$$\langle N_c\rangle = \gamma_c^{(0)}\lambda_c^{(0)}\tilde{N}_1, \quad (16)$$

$$\langle N_{\bar{c}}\rangle = \gamma_c^{(0)}(\lambda_c^{(0)})^{-1}\tilde{N}_{\bar{1}}. \quad (17)$$

The solution can be easily found:

$$\gamma_c^{(0)} = \sqrt{\frac{\langle N_c\rangle\langle N_{\bar{c}}\rangle}{\tilde{N}_1\tilde{N}_{\bar{1}}}}, \quad (18)$$

$$\lambda_c^{(0)} = \sqrt{\frac{\langle N_c\rangle\tilde{N}_{\bar{1}}}{\langle N_{\bar{c}}\rangle\tilde{N}_1}}. \quad (19)$$

Now one can calculate the number of single, hidden, double and triple charm particles in the zero approximation:

$$\langle N_1\rangle_{GCE}^{(0)} = \gamma_c^{(0)}\lambda_c^{(0)}\tilde{N}_1 = \langle N_c\rangle, \quad (20)$$

$$\langle N_H\rangle_{GCE}^{(0)} = (\gamma_c^{(0)})^2\tilde{N}_H = \langle N_c\rangle\langle N_{\bar{c}}\rangle\frac{\tilde{N}_H}{\tilde{N}_1\tilde{N}_{\bar{1}}}, \quad (21)$$

$$\langle N_2\rangle_{GCE}^{(0)} = (\gamma_c^{(0)})^2(\lambda_c^{(0)})^2\tilde{N}_2 = \langle N_c\rangle^2\frac{\tilde{N}_2}{\tilde{N}_1^2}, \quad (22)$$

$$\langle N_3\rangle_{GCE}^{(0)} = (\gamma_c^{(0)})^2(\lambda_c^{(0)})^2\tilde{N}_3 = \langle N_c\rangle^3\frac{\tilde{N}_3}{\tilde{N}_1^3}. \quad (23)$$

The formulas for anticharm can be obtained by the obvious replacement $c \to \bar{c}$, $1 \to \bar{1}$ etc.

The formulas (20–23) are obtained under the assumption that the charm enhancement factor γ_s is not large, so that only a tiny fraction of the charm quarks and antiquarks hadronizes into hidden, double and triple charm particles. This may be not true at very high energies. Indeed, let us consider for example the leading order expression (22) for the number of double charm particles. The fraction of charm quarks that hadronize into double charm particles is proportional to the number of charm quarks in the system and inversely proportional to the system volume at freeze-out:

$$\frac{\langle N_2\rangle_{GCE}^{(0)}}{\langle N_c\rangle} \propto \frac{\langle N_c\rangle}{V}. \quad (24)$$

The volume is proportional to the multiplicity of light hadrons. The charm production cross section grows with the collision energy faster than the multiplicity of light hadrons. Therefore, the ratio (24) becomes comparable to unity at some point. In this case the above approximation does not work.

The accuracy of the leading-order approximation (20)–(23) can be estimated by calculating the next-to-leading order corrections. Let us substitute $\gamma_c \to \gamma_c^{(0)} + \gamma_c^{(1)}$ and $\lambda_c \to \lambda_c^{(0)} + \lambda_c^{(1)}$ into (11) and (12). Neglecting higher order terms like $\gamma_c^{(1)}\lambda_c^{(1)}$, $\gamma_c^{(1)}\tilde{N}_k$ ($k = H, 2, \bar{2}, 3, \bar{3}$), etc. and taking into account (16),(17) and (21)–(23) one gets

$$\lambda_c^{(0)}\gamma_c^{(1)} + \gamma_c^{(0)}\lambda_c^{(1)} = -\frac{\langle N_H\rangle_{GCE}^{(0)} + 2\langle N_2\rangle_{GCE}^{(0)} + 3\langle N_3\rangle_{GCE}^{(0)}}{\tilde{N}_1}, \quad (25)$$

$$\left(\lambda_c^{(0)}\right)^{-2}\left(\lambda_c^{(0)}\gamma_c^{(1)} - \gamma_c^{(0)}\lambda_c^{(1)}\right) = -\frac{\langle N_H\rangle_{GCE}^{(0)} + 2\langle N_{\bar{2}}\rangle_{GCE}^{(0)} + 3\langle N_{\bar{3}}\rangle_{GCE}^{(0)}}{\tilde{N}_{\bar{1}}}. \quad (26)$$

These coupled linear equations can be easily solved:

$$\gamma_c^{(1)} = -\frac{1}{2}\gamma_c^{(0)}\left(\frac{\langle N_H\rangle_{GCE}^{(0)} + 2\langle N_2\rangle_{GCE}^{(0)} + 3\langle N_3\rangle_{GCE}^{(0)}}{\langle N_c\rangle} + \frac{\langle N_H\rangle_{GCE}^{(0)} + 2\langle N_{\bar{2}}\rangle_{GCE}^{(0)} + 3\langle N_{\bar{3}}\rangle_{GCE}^{(0)}}{\langle N_{\bar{c}}\rangle}\right), \quad (27)$$

$$\lambda_c^{(1)} = -\frac{1}{2}\lambda_c^{(0)}\left(\frac{\langle N_H\rangle_{GCE}^{(0)}+2\langle N_2\rangle_{GCE}^{(0)}+3\langle N_3\rangle_{GCE}^{(0)}}{\langle N_c\rangle} - \frac{\langle N_H\rangle_{GCE}^{(0)}+2\langle N_{\bar{2}}\rangle_{GCE}^{(0)}+3\langle N_{\bar{3}}\rangle_{GCE}^{(0)}}{\langle N_{\bar{c}}\rangle}\right). \quad (28)$$

Now the next-to-leading order corrections to the number of particles with different charm content can be calculated:

$$\langle N_1\rangle_{GCE}^{(1)} = -\left(\langle N_H\rangle_{GCE}^{(0)}+2\langle N_2\rangle_{GCE}^{(0)}+3\langle N_3\rangle_{GCE}^{(0)}\right), \quad (29)$$

$$\langle N_H\rangle_{GCE}^{(1)} = \langle N_H\rangle_{GCE}^{(0)}\left(\frac{\langle N_1\rangle_{GCE}^{(1)}}{\langle N_c\rangle}+\frac{\langle N_{\bar{1}}\rangle_{GCE}^{(1)}}{\langle N_{\bar{c}}\rangle}\right), \quad (30)$$

$$\langle N_2\rangle_{GCE}^{(1)} = \langle N_2\rangle_{GCE}^{(0)}\frac{2\langle N_1\rangle_{GCE}^{(1)}}{\langle N_c\rangle}, \quad (31)$$

$$\langle N_3\rangle_{GCE}^{(1)} = \langle N_3\rangle_{GCE}^{(0)}\frac{3\langle N_1\rangle_{GCE}^{(1)}}{\langle N_c\rangle}. \quad (32)$$

The higher order corrections can be obtained in a similar way. Still, the expressions become rather unwieldy. It is more reasonable to solve the coupled equations (11) and (12) numerically, if the next-to-leading order corrections (29)–(32) become too large.

The above formulas allow to calculate the total number of particles with given (anti)charm content (hidden, single, double or triple). If the particle number of a single species is needed, it can be found in the following way. One can deduce from equations (4), (5) and (6) that the number of particles of a single species l can be found from

$$N_l = \frac{\tilde{N}_l}{\tilde{N}_k}N_k, \quad (33)$$

where $k = H, 1, \bar{1}, 2, \bar{2}, 3$ or $\bar{3}$, depending on the charm content of the species l.

The grand canonical approach is the simplest version of the statistical coalescence model. However, it does not reflect the real experimental situation sufficiently well. Indeed, we operated only with average values $\langle N_c\rangle$ and $\langle N_{\bar{c}}\rangle$ in the above consideration. The correlations between N_c and $N_{\bar{c}}$ were ignored. In reality, however, the quarks and antiquarks are produced in pairs. So that not only average numbers coincide $\langle N_c\rangle = \langle N_{\bar{c}}\rangle$, but also the numbers of quarks and antiquarks *in every single event* are equal: $N_c = N_{\bar{c}}$. The influence of this fact on the results of the SCM will be studied in the subsequent sections.

III. SYSTEM WITH FIXED NUMBERS OF CHARM QUARKS AND ANTIQUARKS

In this section, a system with fixed numbers of charm quarks and antiquarks is considered. This does not correspond exactly to what is observed in the experiment. Still a study of such a system would allow us to obtain an important intermediate result. Later, it will be used in more realistic calculations.

Using Eq.(6), one can rewrite the partition function (1) in a more compact form:

$$\mathcal{Z}(V,T,\{\lambda\}) = \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\ldots\sum_{i_L=1}^{\infty}\prod_{l=0}^{L}\frac{N_l^{i_l}}{i_l!} = \exp\left(\sum_{l=0}^{L}N_l\right) = \exp\left(N_{tot}\right). \quad (34)$$

The total number of particles in the last expression can be represented according to (8), then

$$\mathcal{Z}(V,T,\{\lambda\}) = \exp\left(\sum_k N_k\right) = \prod_k \exp\left(N_k\right), \quad (35)$$
$$k = 0, H, 1, \bar{1}, 2, \bar{2}, 3, \bar{3}.$$

Expanding all the exponents in the product, except the first one, into the Taylor series one gets

$$\mathcal{Z}(V,T,\{\lambda\}) = e^{N_0}\sum_{i_H=0}^{\infty}\sum_{i_1=0}^{\infty}\ldots\sum_{i_{\bar{3}}=1}^{\infty}\prod_k\frac{N_k^{i_k}}{i_k!}, \quad (36)$$
$$k = H, 1, \bar{1}, 2, \bar{2}, 3, \bar{3}.$$

Due to the property of The Kronecker delta $\sum_m \delta(m,n) = 1$, nothing changes if we multiply an expression by a Kronecker delta and sum over one of its indices. Doing this twice on the expression (36), one gets

$$\mathcal{Z}(V,T,\{\lambda\}) = e^{N_0}\sum_{i_H=0}^{\infty}\sum_{i_1=0}^{\infty}\ldots\sum_{i_{\bar{3}}=1}^{\infty}\sum_{N_c}\sum_{N_{\bar{c}}}\delta(N_c, i_H+i_1+i_2+i_3)\delta(N_{\bar{c}}, i_H+i_{\bar{1}}+i_{\bar{2}}+i_{\bar{3}})\prod_k\frac{N_k^{i_k}}{i_k!}, \quad (37)$$
$$k = H, 1, \bar{1}, 2, \bar{2}, 3, \bar{3}.$$

Then, after changing the summation order and using (4), (5), (6) and (13), the above expression can be rewritten as

$$\mathcal{Z}(V,T,\{\lambda\}) \;=\; e^{N_0} \sum_{N_c} \sum_{N_{\bar{c}}} \gamma_c^{N_c+N_{\bar{c}}} \exp\left[(N_c - N_{\bar{c}})\frac{\mu_c}{T}\right] Z_{N_c N_{\bar{c}}}(V,T,\{\tilde{\lambda}\}). \tag{38}$$

Here $Z_{N_c N_{\bar{c}}}$ is the partition functions for the systems containing exactly N_c charm quarks and $N_{\bar{c}}$ antiquarks:

$$Z_{N_c N_{\bar{c}}}(V,T,\{\tilde{\lambda}\}) \;=\; \sum_{i_H=0}^{\infty} \sum_{i_1=0}^{\infty} \ldots \sum_{i_{\bar{3}}=1}^{\infty} \delta(N_c, i_H + i_1 + i_2 + i_3)\, \delta(N_{\bar{c}}, i_H + i_{\bar{1}} + i_{\bar{2}} + i_{\bar{3}}) \prod_k \frac{\tilde{N}_k^{i_k}}{i_k!}, \tag{39}$$

$$k = H, 1, \bar{1}, 2, \bar{2}, 3, \bar{3}.$$

This function is *canonical* with respect to the *exact* conservation of the number of charm quarks and antiquarks and *grand canonical* with respect to the conservation *in average* of all other charges, whose values are controlled by the activities

$$\tilde{\lambda}_k \;=\; \lambda_k|_{\mu_c=0,\gamma_c=1}\,. \tag{40}$$

The number of summations in (39) can be reduced due to the Kronecker deltas:

$$Z_{N_c N_{\bar{c}}}(V,T,\{\tilde{\lambda}\}) = \sum_{i_H=0}^{i_H^{max}} \frac{\tilde{N}_H^{i_H}}{i_H!} \sum_{i_2=0}^{i_2^{max}} \frac{\tilde{N}_2^{i_2}}{i_2!} \sum_{i_{\bar{2}}=0}^{i_{\bar{2}}^{max}} \frac{\tilde{N}_{\bar{2}}^{i_{\bar{2}}}}{i_{\bar{2}}!} \sum_{i_3=0}^{i_3^{max}} \frac{\tilde{N}_3^{i_3}}{i_3!} \sum_{i_{\bar{3}}=0}^{i_{\bar{3}}^{max}} \frac{\tilde{N}_{\bar{3}}^{i_{\bar{3}}}}{i_{\bar{3}}!} \frac{\tilde{N}_1^{i_1}}{i_1!} \frac{\tilde{N}_{\bar{1}}^{i_{\bar{1}}}}{i_{\bar{1}}!}. \tag{41}$$

Here

$$i_H^{max} \;=\; min(N_c, N_{\bar{c}}), \tag{42}$$
$$i_2^{max}(i_H) \;=\; [(N_c - i_H)/2], \tag{43}$$
$$i_{\bar{2}}^{max}(i_H) \;=\; [(N_{\bar{c}} - i_H)/2], \tag{44}$$
$$i_3^{max}(i_H, i_2) \;=\; [(N_c - i_H - 2i_2)/3], \tag{45}$$
$$i_{\bar{3}}^{max}(i_H, i_{\bar{2}}) \;=\; [(N_{\bar{c}} - i_H - 2i_{\bar{2}})/3], \tag{46}$$
$$i_1(i_H, i_2, i_3) \;=\; [N_c - i_H - 2i_2 - 3i_3], \tag{47}$$
$$i_{\bar{1}}(i_H, i_{\bar{2}}, i_{\bar{3}}) \;=\; [N_{\bar{c}} - i_H - 2i_{\bar{2}} - 3i_{\bar{3}}]. \tag{48}$$

(The square brackets mean here the integer part, i.e. $[x]$ is the largest integer number that does not exceed x.)

The average number of hidden (single, double, triple) (anti)charm particles in the system with *fixed* numbers of charm quarks and antiquarks is found as

$$\langle N_k \rangle_{fix} = \sum_l \tilde{\lambda}_l \frac{\partial \log Z_{N_c N_{\bar{c}}}}{\partial \tilde{\lambda}_l} = \sum_l \tilde{\lambda}_l \frac{\partial \tilde{N}_k}{\partial \tilde{\lambda}_l} \frac{\partial \log Z_{N_c N_{\bar{c}}}}{\partial \tilde{N}_k},$$
$$k = H, 1, \bar{1}, 2, \bar{2}, 3, \bar{3}; \tag{49}$$

l runs over all hadron species of the type k.

From (6) one sees that

$$\tilde{\lambda}_l \frac{\partial \tilde{N}_l}{\partial \tilde{\lambda}_l} = \tilde{N}_l, \tag{50}$$

therefore

$$\sum_l \tilde{\lambda}_l \frac{\partial \tilde{N}_k}{\partial \tilde{\lambda}_l} = \tilde{N}_k \tag{51}$$

and finally

$$\langle N_k \rangle_{fix} = \tilde{N}_k \frac{1}{Z_{N_c N_{\bar{c}}}} \frac{\partial Z_{N_c N_{\bar{c}}}}{\partial \tilde{N}_k}. \tag{52}$$

I restrict my further consideration to the case, when the partition function (41) is dominated[3] by the term with $i_H = i_2 = i_{\bar{2}} = i_3 = i_{\bar{3}} = 0$, $i_1 = N_c$ and $i_{\bar{1}} = N_{\bar{c}}$:

$$Z_{N_c N_{\bar{c}}}^{(0)} \approx \frac{\tilde{N}_1^{N_c}}{N_c!} \frac{\tilde{N}_{\bar{1}}^{N_{\bar{c}}}}{N_{\bar{c}}!}. \tag{53}$$

The same term dominates also the derivatives of $Z_{N_c N_{\bar{c}}}$ with respect to N_c and $N_{\bar{c}}$. It is easy to see that in this case, most of the charm hadronizes into hadrons containing only one c-quark or antiquark:

$$\langle N_1 \rangle_{fix}^{(0)} \;\approx\; N_c, \tag{54}$$
$$\langle N_{\bar{1}} \rangle_{fix}^{(0)} \;\approx\; N_{\bar{c}}. \tag{55}$$

Only a tiny fraction of the total charm is accommodated into hidden, double and triple charm hadrons.

The leading term (53) does not depend on i_H. Therefore, the derivative $\partial Z_{N_c N_{\bar{c}}}/\partial \tilde{N}_H$ is dominated by the term of (41) with $i_H = 1$, $i_2 = i_{\bar{2}} = i_3 = i_{\bar{3}} = 0$, $i_1 = N_c - 1$ and $i_{\bar{1}} = N_{\bar{c}} - 1$:

$$\frac{\partial Z_{N_c N_{\bar{c}}}^{(0)}}{\partial \tilde{N}_H} \approx \frac{\tilde{N}_1^{N_c-1}}{(N_c-1)!} \frac{\tilde{N}_{\bar{1}}^{N_{\bar{c}}-1}}{(N_{\bar{c}}-1)!}. \tag{56}$$

[3] Due to the inequality (15), the contribution of other terms becomes sizable only at very large N_c and/or $N_{\bar{c}}$. This case can be treated within the grand canonical approach and there is no reason to study it here.

From (52) one finds the average number of hidden charm particles in the system with fixed numbers of charm quarks and antiquarks:

$$\langle N_H \rangle_{fix}^{(0)} = N_c N_{\bar{c}} \frac{\tilde{N}_H}{\tilde{N}_1 \tilde{N}_{\bar{1}}}. \tag{57}$$

Similarly, the average numbers of double and triple charm particles are given by:

$$\langle N_2 \rangle_{fix}^{(0)} = N_c (N_c - 1) \frac{\tilde{N}_2}{\tilde{N}_1^2} \tag{58}$$

and

$$\langle N_3 \rangle_{fix}^{(0)} = N_c (N_c - 1)(N_c - 2) \frac{\tilde{N}_3}{\tilde{N}_1^3}, \tag{59}$$

respectively. The corresponding formulas for anticharm can be obtained from (58) and (59) by the obvious replacement $c \to \bar{c}$, $1 \to \bar{1}$ etc.

We have considered the charm hadron production by quark coalescence in the thermal hadron system with *fixed* numbers of charm quarks and antiquarks. In reality, this number is not fixed. It fluctuates from event to event. These fluctuations have to be taken into account in more realistic calculations.

IV. THE SYSTEM WITH POISSONIAN FLUCTUATIONS OF THE NUMBER OF CHARM QUARK-ANTIQUARK PAIRS

In a relativistic collision of two nuclei, the charm quarks and antiquarks are created in pairs in independent nucleon-nucleon collisions. Therefore, the number of quarks in the system is always equal to the number of antiquarks:

$$N_c = N_{\bar{c}} \equiv N_{c\bar{c}} \tag{60}$$

and the fluctuations of the number of pairs approximately[4] conforms the Poissonian law:

$$w_P(N_{c\bar{c}}) = e^{-\langle N_{c\bar{c}} \rangle} \frac{\langle N_{c\bar{c}} \rangle^{N_{c\bar{c}}}}{N_{c\bar{c}}!}. \tag{61}$$

Here $w(N_{c\bar{c}})$ is the probability to observe $N_{c\bar{c}}$ charm quark-antiquark pairs in an event, provided that the average number of $c\bar{c}$ pairs in this type of events is $\langle N_{c\bar{c}} \rangle$.

The average number of particles with hidden, single, double, and triple charm is found by the convolution of the results of the last section (54),(57)–(59) with the probability (61):

$$\langle N_k \rangle_P = \sum_{N_{c\bar{c}}=1}^{\infty} w_P(N_{c\bar{c}}) \langle N_k \rangle_{fix} \tag{62}$$
$$k = H, 1, \bar{1}, 2, \bar{2}, 3, \bar{3}$$

(The subscript "P" stands for "Poisson"). Here we again see that the most of the charm is accommodated into the single charm hadrons:

$$\langle N_1 \rangle_P^{(0)} \approx \langle N_{\bar{1}} \rangle_P^{(0)} \approx \langle N_{c\bar{c}} \rangle. \tag{63}$$

The rest (a tiny fraction) is distributed over hidden, double and triple charm particles whose number can be easily found:

$$\langle N_H \rangle_P^{(0)} = \langle N_{c\bar{c}} \rangle (\langle N_{c\bar{c}} \rangle + 1) \frac{\tilde{N}_H}{\tilde{N}_1 \tilde{N}_{\bar{1}}}, \tag{64}$$

$$\langle N_2 \rangle_P^{(0)} = \langle N_{c\bar{c}} \rangle^2 \frac{\tilde{N}_2}{\tilde{N}_1^2}, \tag{65}$$

$$\langle N_3 \rangle_P^{(0)} = \langle N_{c\bar{c}} \rangle^3 \frac{\tilde{N}_3}{\tilde{N}_1^3}. \tag{66}$$

Surprisingly, the formulas for the number of double and triple charm particles (65),(66) appeared to be exactly the same as in the grand canonical approach (22),(23).

However, this is not the case for the hidden charm. If the average number of $c\bar{c}$ pairs is small, $\langle N_{c\bar{c}} \rangle \lesssim 1$, the average number of produced hidden charm hadrons (64) is essentially larger than it would be naïvely expected from the grand canonical formula (21). The two formulas (21) and (64) give similar results only when the number $c\bar{c}$ pairs is large, $\langle N_{c\bar{c}} \rangle \gg 1$. The reason for this similarity is that the Poissonian distribution becomes narrow at $\langle N_{c\bar{c}} \rangle \gg 1$: $\langle (N_{c\bar{c}} - \langle N_{c\bar{c}} \rangle)^2 \rangle = \langle N_{c\bar{c}} \rangle \ll \langle N_{c\bar{c}} \rangle^2$.

It is easy to see that any narrow probability distribution of the number of charm quarks and antiquarks would give the same result as the grand canonical approach, provided that $\langle N_{c\bar{c}} \rangle \gg 1$.

V. CHARM COALESCENCE IN A SUBSYSTEM

The formulas of the previous section allow to calculate the number of charm particles in the entire system. It often happens, however, that only a part of the phase space is observed in the experiment: a limited rapidity interval, for instance. Formulas for charm production in a subsystem are necessary in this case. The charm and non-charm hadrons are distributed inhomogeneously in the (phase) space. Therefore, the number of observed charm particles in a subsystem cannot be calculated merely as a fraction of their total number in the system, proportional to the ratio of the volume of the subsystem to the total volume of the system. Moreover, the total volume of the system may be even unknown. The formulas derived in

[4] This approximation obviously breaks down at very large $N_{c\bar{c}}$ when the energy of the produced $c\bar{c}$ pairs becomes comparable with the total energy of the system. But the probability of such events is clearly negligible. Only a tiny fraction of the total energy of the system is accumulated into the charm particles.

this section are based on the thermodynamic parameters of the subsystem without any reference to those of the entire system.

Let $\xi \le 1$ is the probability to find a charm quark in the subsystem, provided that exactly one $c\bar{c}$ pair is present in the entire system. Then, if the number of $c\bar{c}$ pairs in the entire system is $N_{c\bar{c}}$, the probability to find N_c charm quarks in the subsystem is given by the binomial law:

$$w(N_c|N_{c\bar{c}}) = \frac{N_{c\bar{c}}!}{N_c!(N_{c\bar{c}} - N_c)!} \xi^{N_c} (1-\xi)^{N_{c\bar{c}} - N_c}. \quad (67)$$

It is assumed that the distributions of quarks and antiquarks are uncorrelated, and the probability distribution of the number of antiquarks $N_{\bar{c}}$ is given by the same binomial law.

Then, if the total number of $c\bar{c}$ pairs in the system is fixed, the average number of the hidden charm particles produced in the system is given by a convolution of two probability distributions (67) with the right-hand side of (57):

$$\begin{aligned} \langle N_H \rangle^{(0)}_{fix} &= \sum_{N_c=0}^{\infty} w(N_c|N_{c\bar{c}}) \sum_{N_c=0}^{\infty} w(N_{\bar{c}}|N_{c\bar{c}}) N_c N_{\bar{c}} \frac{\tilde{N}_H}{\tilde{N}_1 \tilde{N}_{\bar{1}}} \\ &= \xi^2 (N_{c\bar{c}})^2 \frac{\tilde{N}_H}{\tilde{N}_1 \tilde{N}_{\bar{1}}}. \end{aligned} \quad (68)$$

In a more realistic situation, if the total number of $c\bar{c}$ pairs in the entire system fluctuates according to the Poissonian law (61), the average number of hidden charm in the subsystem is given by

$$\langle N_H \rangle^{(0)}_{sub} = \xi^2 \langle N_{c\bar{c}} \rangle (\langle N_{c\bar{c}} \rangle + 1) \frac{\tilde{N}_H}{\tilde{N}_1 \tilde{N}_{\bar{1}}}. \quad (69)$$

Similarly for the double and triple charm:

$$\langle N_2 \rangle^{(0)}_{sub} = \xi^2 \langle N_{c\bar{c}} \rangle^2 \frac{\tilde{N}_2}{\tilde{N}_1^2}, \quad (70)$$

$$\langle N_3 \rangle^{(0)}_{sub} = \xi^3 \langle N_{c\bar{c}} \rangle^3 \frac{\tilde{N}_3}{\tilde{N}_1^3}. \quad (71)$$

Note, that only the average number of the charm quark-antiquark pairs refers to the entire system in the above equations. The thermal quantities N_H, $\tilde{N}_1$ and $\tilde{N}_{\bar{1}}$ are related to the subsystem under consideration. Therefore, one can calculate the number of charm particles produced in the subsystem, even if the thermodynamic parameters of the entire system are not known. Moreover, to calculate the number of double and triple charm, one does not actually need the total number of $c\bar{c}$ pairs in the entire system. It suffices to know this number for the subsystem $\xi \langle N_{c\bar{c}} \rangle$. Only the calculation of the hidden charm requires the total number of charm pairs in the entire system, if this number is comparable to 1 or smaller.

VI. THE CANONICAL APPROACH AND WHY IT IS NOT APPROPRIATE FOR THE CHARM COALESCENCE

The canonical ensemble (CE) approach was initially proposed for a thermal treatment of the strangeness productions, when the average number of the strange particles in the system is small ($\lesssim 1$) and the exact conservation of the net strangeness becomes important [17]. It was also applied to the baryonic charge [18].

Although, as it will be explained later, this approach is not appropriate for the charm coalescence, I consider it in details because of two reasons. First, it has been widely used for the description of the charmonium production [6, 7, 12, 19]. Second, it would be instructive to see, what changes, if the event-by-event fluctuations of the number of charm pairs is different from the Poissonian one.

From the formal point of view, there is no problem to apply the canonical approach to the charm coalescence. Let us choose the probability distribution of the event-by-event fluctuations of the number of $c\bar{c}$ pairs as

$$w_{CE}(N_{c\bar{c}}) = \frac{\gamma_c^2 Z_{N_{c\bar{c}}}(V, T, \{\tilde{\lambda}\})}{\sum_{N_{c\bar{c}}=0}^{\infty} \gamma_c^{2N_{c\bar{c}}} Z_{N_{c\bar{c}}}(V, T, \{\tilde{\lambda}\})}, \quad (72)$$

where

$$Z_{N_{c\bar{c}}}(V, T, \{\tilde{\lambda}\}) \equiv Z_{N_c N_{\bar{c}}}(V, T, \{\tilde{\lambda}\}) \Big|_{N_c = N_{\bar{c}} \equiv N_{c\bar{c}}}. \quad (73)$$

In the zero approximation (53), the sum in the denominator can be expressed via the modified Bessel function I_0:

$$\begin{aligned} \sum_{N_{c\bar{c}}=0}^{\infty} \gamma_c^{2N_{c\bar{c}}} Z_{N_{c\bar{c}}} &\approx \sum_{N_{c\bar{c}}=0}^{\infty} \frac{\left(\gamma_c^2 \tilde{N}_1 \tilde{N}_{\bar{1}}\right)^{N_{c\bar{c}}}}{(N_{c\bar{c}}!)^2} \\ &= I_0\left(2\gamma_c \sqrt{\tilde{N}_1 \tilde{N}_{\bar{1}}}\right). \end{aligned} \quad (74)$$

Again, to find the number of charm particles with different charm content we have to convolute the probability (72) with the expressions (54) and (57)–(59) similarly to (62). The zero approximation results are expressed via the modified Bessel functions I_k, $k = 0, \ldots, 3$:

$$\begin{aligned} \langle N_1 \rangle^{(0)}_{CE} &\approx \langle N_{\bar{1}} \rangle^{(0)}_{CE} \approx \langle N_{c\bar{c}} \rangle \quad (75) \\ &\approx \gamma_c^{(0)} \sqrt{\tilde{N}_1 \tilde{N}_{\bar{1}}} \frac{I_1\left(2\gamma_c^{(0)} \sqrt{\tilde{N}_1 \tilde{N}_{\bar{1}}}\right)}{I_0\left(2\gamma_c^{(0)} \sqrt{\tilde{N}_1 \tilde{N}_{\bar{1}}}\right)}, \end{aligned}$$

$$\langle N_H \rangle^{(0)}_{CE} = \left(\gamma_c^{(0)}\right)^2 \tilde{N}_H, \quad (76)$$

$$\langle N_2 \rangle^{(0)}_{CE} = \left(\gamma_c^{(0)}\right)^2 \tilde{N}_2 \frac{\tilde{N}_{\bar{1}}}{\tilde{N}_1} \frac{I_2\left(2\gamma_c^{(0)} \sqrt{\tilde{N}_1 \tilde{N}_{\bar{1}}}\right)}{I_0\left(2\gamma_c^{(0)} \sqrt{\tilde{N}_1 \tilde{N}_{\bar{1}}}\right)}, \quad (77)$$

$$\langle N_3\rangle_{CE}^{(0)} = \left(\gamma_c^{(0)}\right)^3 \tilde{N}_3 \left(\sqrt{\frac{\tilde{N}_{\bar{1}}}{\tilde{N}_1}}\right)^3 \frac{I_3\left(2\gamma_c^{(0)}\sqrt{\tilde{N}_1\tilde{N}_{\bar{1}}}\right)}{I_0\left(2\gamma_c^{(0)}\sqrt{\tilde{N}_1\tilde{N}_{\bar{1}}}\right)}. \tag{78}$$

The formula (76) for the hidden charm in CE has the same form as the corresponding formula in GCE approach (see the middle part of (21)), but the result is different, because the value of $\gamma_c^{(0)}$ is not the same. Indeed, this value has to be found at given $N_{c\bar{c}}$ from the transcendental equation (75). Its solution may be quite different from (18).

The equation (75) can be solved analytically in two limiting cases: $\langle N_{c\bar{c}}\rangle \gg 1$ and $\langle N_{c\bar{c}}\rangle \ll 1$. In the first case, the ratios of the Bessel functions tends to 1:

$$\frac{I_k\left(x\right)}{I_0\left(x\right)} \simeq 1 \quad \text{at} \quad x \gg 1,\ k = 1, 2, \ldots \tag{79}$$

and the formulas are reduced to those of the grand canonical approach.

In the second case, $\langle N_{c\bar{c}}\rangle \ll 1$, the Bessel functions can be replaced by the leading terms of their Taylor expansions:

$$I_k\left(x\right) \simeq \frac{x^k}{2^k k!} \quad \text{at} \quad x \ll 1,\ k = 0, 2, \ldots . \tag{80}$$

The equation (75) then becomes

$$\langle N_{c\bar{c}}\rangle \simeq \left(\gamma_c^{(0)}\right)^2 \tilde{N}_1\tilde{N}_{\bar{1}}. \tag{81}$$

The factor $\gamma_c^{(0)}$ can be easily found:

$$\gamma_c^{(0)} \simeq \sqrt{\frac{\langle N_{c\bar{c}}\rangle}{\tilde{N}_1\tilde{N}_{\bar{1}}}}. \tag{82}$$

Then the number of hidden, double and triple charm particles can be calculated:

$$\langle N_H\rangle_{CE}^{(0)} \simeq \langle N_{c\bar{c}}\rangle \frac{\tilde{N}_H}{\tilde{N}_1\tilde{N}_{\bar{1}}}, \tag{83}$$

$$\langle N_2\rangle_{CE}^{(0)} \simeq \frac{1}{2}\langle N_{c\bar{c}}\rangle^2 \frac{\tilde{N}_2}{\tilde{N}_1^2}, \tag{84}$$

$$\langle N_3\rangle_{CE}^{(0)} \simeq \frac{1}{6}\langle N_{c\bar{c}}\rangle^3 \frac{\tilde{N}_3}{\tilde{N}_1^3}. \tag{85}$$

The result (83) for the hidden charm coincides with that for the Poissonian fluctuation case (64) in the limit $\langle N_{c\bar{c}}\rangle \ll 1$. The reason for this coincidence is the following property of the both distributions:

$$w(n) \ll w(1) \quad \text{at} \quad \langle N_{c\bar{c}}\rangle \ll 1, \quad n = 2, 3, \ldots . \tag{86}$$

Therefore, the hidden charm hadrons are mostly produced in the systems containing a single $c\bar{c}$ pair. The probabilities to observe exactly one $c\bar{c}$ pair becomes approximately equal to each other for such distributions, if the average number of the pairs is the same and is small:

$$w_P(1) \simeq w_{CE}(1) \simeq \langle N_{c\bar{c}}\rangle \quad \text{at} \quad \langle N_{c\bar{c}}\rangle \ll 1. \tag{87}$$

In contrast, the double and triple charm particles cannot be produced in the system containing only one $c\bar{c}$ pair. Two and three pairs at least are needed to produce, respectively, a double and a triple charm particle. The probability to have two or three pairs in the system are different in the Poissonian and the canonical cases. From this reason, the results are essentially different.

Although the Poissonian and canonical probability laws give the same result for the hidden charm in two limiting cases: $\langle N_{c\bar{c}}\rangle \ll 1$ and $\langle N_{c\bar{c}}\rangle \gg 1$, they are up to about 10% different in the intermediate region $\langle N_{c\bar{c}}\rangle \sim 1$ [6].

Which of two approaches, the Poissonian or the Canonical one, is appropriate for the description of the charm coalescence in a system containing a small number of $c\bar{c}$ pairs? In fact, the key assumption (72) of the canonical approach relates the fluctuations of the number of charm pairs to the thermodynamic parameters of the system at chemical freeze-out. This is in an obvious contradiction with the basic postulates of the statistical coalescence model. Indeed, we have postulated that charm quarks are produced exclusively at the initial stage of the reaction in hard parton collisions. Creation and annihilation at later stages are neglected. Therefore, the fluctuations of the number of the $c\bar{c}$ pairs cannot have any relation to the properties of the system at the thermal stage. From these reasons, the canonical approach is not appropriate for the treatment of charm coalescence.

The case of strangeness hadronization is essentially different: strange quarks and antiquarks can be produced at the thermal stage, as far as the temperature is larger or comparable to the strange quark mass. Therefore, in the case of full strangeness thermalization (if not only the momenta, but the number of strange quark pairs is thermal), the canonical approach is the most appropriate. Still, it should be used with care if the full strangeness thermalization is not reached: $\gamma_s \neq 1$.

VII. SUMMARY AND OUTLOOK

The production of particles with single, double, triple and hidden charm in the framework of the statistical coalescence model has been considered. The grand canonical approach (Section II) is appropriate for systems containing a large number of charm quark-antiquark pairs: $\langle N_{c\bar{c}}\rangle \gg 1$. The solution can be found analytically (20)–(23), if the number of hidden, double and triple charm particles is small comparing to the total charm. This is the case at presently available collision energies. At higher energies, the result can be found numerically from the coupled equations (11) and (12).

The grand canonical approach cannot be applied to the system containing a small number of charm quark-antiquark pairs: $\langle N_{c\bar{c}}\rangle \lesssim 1$. It has been shown that the canonical approach is also inappropriate in this case. It is in variance with the basic postulates of the statistical coalescence model. As far as $c\bar{c}$ pairs are created in mutually independent nucleon-nucleon collisions, the fluctuations of $N_{c\bar{c}}$ follows the Poissonian law. In this case, the result coincides with the grand canonical one for the double and triple charm (65),(66), but differs essentially for the number of the hidden charm (64).

The charm coalescence in a part of a large system has been also studied. It is sufficient to know the thermal parameters of the subsystem under consideration to find the number of particles with different charm content. The thermal properties of the entire system are not needed. Still one has to know the total number of the charm pairs in the entire system to calculate the number of hidden charm particles in the subsystem. For double and triple charm, the information on the number of $c\bar{c}$ pairs in the subsystem suffices.

The obtained derived allow to obtain predictions for double, triple and hidden charm production in heavy ion collisions at all collision energies. At very high energies, like those of the Large Hadron Collider (LHC) in CERN (Switzerland), the expected number of produced charm quark-antiquarks is large (of order of hundreds or even more). Under these conditions, a sizable number of double and triple charm is expected. On the other hand, it would be interesting to see, whether the statistical coalescence model works at low energies. The possibility to study this at the accelerator facility with a very high luminosity which is planned to be build in GSI (Germany) is worth to be checked. A thermal or nonthermal behavior of heavy quarks can tell us much about the properties of the medium created during the heavy ion collision [20].

Acknowledgments

I acknowledge the financial support of the Deutsche Forshunggemanschaft (DFG), Germany.

---

[1] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B **465**, 15 (1999) [arXiv:nucl-th/9903010];
G. D. Yen and M. I. Gorenstein, Phys. Rev. C **59**, 2788 (1999) [arXiv:nucl-th/9808012];
F. Becattini *et al.*, Phys. Rev. C **64**, 024901 (2001) [arXiv:hep-ph/0002267];
W. Florkowski, W. Broniowski and M. Michalec, Acta Phys. Polon. B **33**, 761 (2002) [arXiv:nucl-th/0106009].

[2] F. Becattini and U. W. Heinz, Z. Phys. C **76**, 269 (1997) [Erratum-ibid. C **76**, 578 (1997)] [arXiv:hep-ph/9702274].

[3] F. Becattini, Z. Phys. C **69**, 485 (1996).

[4] M. Gaździcki and M. I. Gorenstein, Phys. Rev. Lett. **83**, 4009 (1999) [hep-ph/9905515].

[5] P. Braun-Munzinger and J. Stachel, Phys. Lett. B **490**, 196 (2000) [arXiv:nucl-th/0007059].

[6] M. I. Gorenstein, A. P. Kostyuk, H. Stöcker and W. Greiner, Phys. Lett. B **509**, 277 (2001) [arXiv:hep-ph/0010148].

[7] M. I. Gorenstein, A. P. Kostyuk, H. Stöcker and W. Greiner, J. Phys. G **27**, L47 (2001) [arXiv:hep-ph/0012015];
P. Braun-Munzinger and J. Stachel, Nucl. Phys. A **690**, 119c (2001) [arXiv:nucl-th/0012064].

[8] A. P. Kostyuk, M. I. Gorenstein, H. Stöcker and W. Greiner, Phys. Lett. B **531**, 195 (2002) [arXiv:hep-ph/0110269]. J. Phys. G **28**, 2297 (2002) [arXiv:hep-ph/0204180];
A. P. Kostyuk, J. Phys. G **28**, 2047 (2002) [arXiv:hep-ph/0111096]; Nucl. Phys. A **715**, 569 (2003) [arXiv:hep-ph/0209139]; arXiv:hep-ph/0306123.

[9] A. Kostyuk and H. Stoecker, GSI Scientific Report 2003, p. 86 [arXiv:nucl-th/0501077].

[10] L. Grandchamp and R. Rapp, Phys. Lett. B **523**, 60 (2001) [arXiv:hep-ph/0103124]; Nucl. Phys. A **709**, 415 (2002) [arXiv:hep-ph/0205305];
L. Grandchamp, R. Rapp and G. E. Brown, Phys. Rev. Lett. **92**, 212301 (2004) [arXiv:hep-ph/0306077].

[11] M. I. Gorenstein, A. P. Kostyuk, H. Stöcker and W. Greiner, Phys. Lett. B **524**, 265 (2002) [arXiv:hep-ph/0104071];
M. I. Gorenstein, A. P. Kostyuk, L. McLerran, H. Stöcker and W. Greiner, J. Phys. G **28**, 2151 (2002) [arXiv:hep-ph/0012292];
A. P. Kostyuk, M. I. Gorenstein, H. Stöcker and W. Greiner, Phys. Rev. C **68**, 041902 (2003) [arXiv:hep-ph/0305277].

[12] A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Phys. Lett. B **571**, 36 (2003) [arXiv:nucl-th/0303036]; Nucl. Phys. A **715**, 529 (2003) [arXiv:nucl-th/0209035].

[13] E. L. Bratkovskaya, A. P. Kostyuk, W. Cassing and H. Stöcker, Phys. Rev. C **69**, 054903 (2004) [arXiv:nucl-th/0402042].

[14] M. A. Moinester, Z. Phys. A **355**, 349 (1996) [arXiv:hep-ph/9506405];
V. V. Kiselev and A. K. Likhoded, Phys. Usp. **45**, 455 (2002) [arXiv:hep-ph/0103169].

[15] M. Mattson *et al.* [SELEX Collaboration], Phys. Rev. Lett. **89**, 112001 (2002) [arXiv:hep-ex/0208014];
A. Ocherashvili *et al.* [SELEX Collaboration], arXiv:hep-ex/0406033.

[16] V. V. Kiselev and A. K. Likhoded, arXiv:hep-ph/0208231.

[17] K. Redlich and L. Turko, Z. Phys. C **5**, 201 (1980);
J. Rafelski and M. Danos, Phys. Lett. B **97**, 279 (1980);
J. Cleymans, K. Redlich and E. Suhonen, Z. Phys. C **51**, 137 (1991).

[18] M. I. Gorenstein, M. Gazdzicki and W. Greiner, Phys. Lett. B **483**, 60 (2000) [arXiv:hep-ph/0001112].

[19] N. Brambilla *et al.*, arXiv:hep-ph/0412158.
[20] Y. L. Dokshitzer and D. E. Kharzeev, Phys. Lett. B **519**, 199 (2001) [arXiv:hep-ph/0106202]; H. van Hees and R. Rapp, arXiv:nucl-th/0412015.